FINITE ELEMENT ANALYSES AND COFFERDAM EXCAVATIONS

Dimitrios C. Konstantakos, Mueser Rutledge Consulting Engineers, New York, NY, USA

Cofferdam excavations have traditionally been designed with classical design methods that involve direct computations of active or apparent earth pressures. However in contrast to finite element methods, traditional analyses can not account for full soil-structure interaction. Because of their complexity, numerical analyses are performed almost exclusively by computers. Recent theoretical and computer advancements have made the use of the finite element method much more user-friendly and affordable. General information about the basics of the finite element method in geotechnical engineering and for cofferdam excavations is presented. While knowledge of all the intricate theoretical and computational details is not needed, the designer should at a minimum be aware of the most important aspects of soil models. Most importantly, the designer must have sufficient experience to be able to judge the relative "correctness" of vast numerical results produced by today's software programs. Hence, the human factor is an irreplaceable component of finite element computations.

Introduction

Cofferdams are enclosed temporary excavations that retain soil and water outside of a construction site. Marine cofferdams typically provide temporary excavation support for construction of bridge piers or other structures that have to be founded a considerable depth below water. In the majority of cases, marine cofferdams comprise a system of joining steel sheet piles braced by steel members.

Cofferdam excavations have been in use for a long time. In most projects, cofferdams have been designed with conventional analysis methods, where the soil pressure distribution is based on limit equilibrium theory (active – passive) or on empirical pressure distributions (Peck 1969 and others). The major disadvantage of "classical" approaches is that the relation between soil pressures, wall displacements, and construction staging can not be captured. Here lies the "beauty" of finite element methods. Literally, the finite element method is a divide and conquer approach to solving engineering problems. This is achieved by descretizing the soil continuum and structural elements in basic elements where a known mathematical framework can be applied to assure force equilibrium and displacement consistency. While performing such calculations and setting the proper finite element mathematical framework with hand calculations is possible, the task would be so daunting that an engineer could well spend his entire career solving one problem! Hence, finite element problems have been traditionally solved with the aid of a computer. Recent advancements in computer technology and geotechnical modeling have made the use of finite elements a very effective method for analyzing cofferdam excavations.

While in the majority of cofferdams it is not necessary to conduct finite element analyses, it would be beneficial to perform analyses in projects that require or involve any of the following:

a. Accurate determination of lateral and vertical soil and wall displacements.
b. Construction adjacent to sensitive structures.
c. Basal stability in soft clays.
d. Accurate groundwater modeling.
e. Unbalanced soil loading conditions.
f. Complicated soil profile and site conditions.
g. Consideration of construction staging effects.
h. Full accountability of construction staging.
i. Consolidation effects.
j. Considerable excavation depth (40 ft or more)
k. Three dimensional effects.

It would be impossible and impractical to cover every aspect of geotechnical finite element modeling in this publication. Hence, the author aims to provide the reader with a basic understanding and a general approach of how finite element theory can be used to analyze cofferdam excavations.

Finite Element Basics

The finite element method is a general systematic computational procedure for approximately solving problems in physics and engineering. The term finite element method was coined in the late 1950s to define the method for computer- based structural analyses.

In geotechnical engineering, the finite element method involves dividing the soil and structures into discrete elements that are easier to analyze. Then, material properties and behavior are described by a set of predefined models. Equilibrium and energy numerical approximations are then used to simplify the analysis and satisfy all necessary conditions for equilibrium. Thus by the nature of the method, it is unavoidable that all FE analyses will contain a certain degree of numerical error.

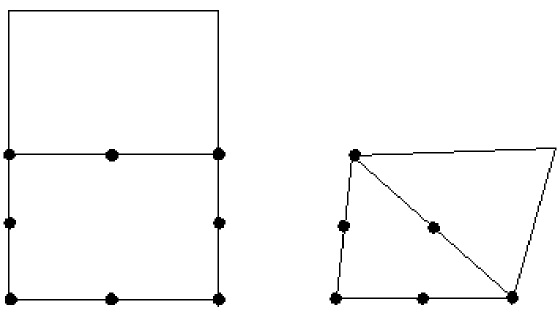


Figure 1: Rectangular and triangular finite elements

The most used types of discretized elements are rectangular and triangular elements as shown in Figure 1. The governing equilibrium equations are always performed in matrix operations as demonstrated in Figure 2. However, for more complicated element types the matrix equations can become extremely complicated and time consuming. While matrix operations are daunting to most engineers (including the author), the beauty of recent software developments is that the end user never has to deal with the intricate details of each step by step matrix operation. Thus, research centers and major finite element software organizations have effectively taken the computational burden off the shoulders of practicing engineers. However, at a minimum, a user should be aware of the general manner in which the finite element method is applied by the software program he or she decides to use because in this day and age, it is almost unimaginable for anybody to attempt solving a finite element problem by hand computations.

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xx} \\ \gamma_{yy} \\ \gamma_{zz} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -v & -v & 0 & 0 & 0 \\ -v & 1 & -v & 0 & 0 & 0 \\ -v & -v & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+v) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+v) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+v) \end{bmatrix} \begin{bmatrix} \sigma'_{xx} \\ \sigma'_{yy} \\ \sigma'_{zz} \\ \sigma'_{xy} \\ \sigma'_{yz} \\ \sigma'_{zx} \end{bmatrix}$$

E= Young's modulus v= Poisson's ratio

σ'_{xx}= Effective stress

Figure 2: Typical Matrix Equilibrium Set Up.

<u>Soil Behavior & Stiffness Modeling</u>

One of the major challenges in geotechnical engineering and soil structure interaction is the “true” modeling of soil behavior. In this continuing effort, many researchers have proposed various soil models (Ref:). Soil models basically comprise mathematical frameworks that relate parameters such as elasticity, stress, displacement, time, failure criteria, and stress history. Most idealized soil behavior can be described by a response included in Figure 3. Most soil models used in finite element analyses generally fall under or combine some of the following categories:

I. Linear elastic soil behavior (strain increases linearly with stress).
II. Linearly elastic-perfectly plastic (strain increases linearly with stress until yielding occurs. Beyond yielding, stress remains the same at increasing strain levels i.e. plastic behavior). These models offer a simple idealization of soil behavior; however, they may not capture correctly displacements near yielding.
III. Non linear models with strain hardening or strain softening behavior. Almost all soil models used in practice can not capture strain softening behavior.
IV. Time dependent features for consolidation and creep. Secondary displacements associated with long term soil creep may have to be considered when modeling soft clays. Soil creep and consolidation are typically not significant issues in temporary works such as cofferdam excavations that are constructed within a short time span.
V. Different loading and unloading behavior. Overconsolidated soils are typically stiffer when loaded or unloaded to smaller stress levels than the maximum soil stress state experienced in the past. In this region, soils are slightly stiffer in the unloading mode than in the loading mode (Figure 3). All soil models known to the author that include unloading behavior, model both loading and unloading response as linear (in the unloaded region). This behavior is a good approximation when the load is not cycled between two stress

levels a significant number of times. Unfortunately, no rational framework has been developed to properly model plastic soil straining from load cycling.

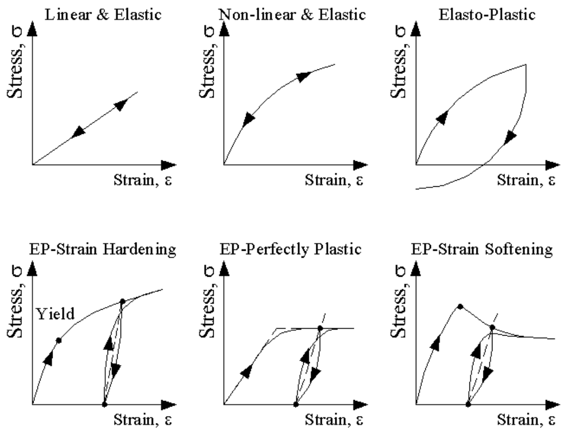
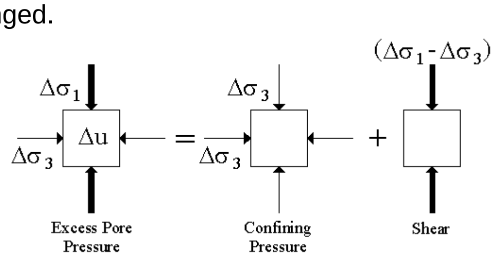


Figure 3: Idealized Modes of Elastic Soil Behavior (Whittle, 2005)

Soil Strength

An equally important aspect of capturing soil behavior is defining proper failure and yielding criteria. Natural soils Figure considerably more complicated yielding behavior than engineering materials such as steel. Traditionally, soil strength has been described by a cohesive strength and an internal frictional angle (Mohr, 1772). Depending on the soil type, soil strength is strongly on the rate of loading (or unloading). For purely frictional soils (sands) the rate of loadings has almost no effect (with the exception of intense earthquakes).

On the other hand, clays display different strength for short and long term loadings. The short term loading strength behavior is also routinely referred to as "undrained" strength behavior, and has been described by Skempton (1951) and others (Figure 4). Because of its low porosity and internal crystal electrical ionization forces, a clay soil does not allow pore water to easily escape (i.e. drain) when it experiences a "fast" load change. As a result and because water is practically incompressible, excess internal pore pressures develop that resist applied load changes. Hence, initially there is little change in effective stress while the total stress level has changed.



$$\Delta u = B\,\Delta\sigma_3 + B\;A\;(\Delta\sigma_1 - \Delta\sigma_3)$$

Figure 4: Pore Pressure Parameters (Skempton, 1951)

Clay strength is also influenced by the direction of loading (or loading mode). Naturally, clays Figure greater undrained strength in compression than in tension. When basal movements are involved, clays may be compressed, sheared, and stretched as shown in Figure 5. Unfortunately, this strength anisotropy may not be properly captured by a single set of parameters for most soil models. If anisotropy plays a significant role, then one may have to specify different soil strengths for the same soil layer that describe the different loading mode strengths. Unified advanced soil models such as the MIT-E3 soil model can account for soil anisotropy (Kavvadas, 1989)

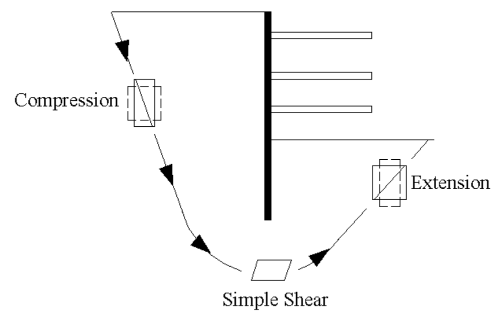


Figure 5: Models of soil behavior in an excavation with basal movements.

Typical Advanced Soil Models

Capturing the "true" soil behavior is a significant challenge. While simple linear-elastic-perfectly plastic models can produce reasonable first order results, a more realistic response can be obtained by simulating soil as a non-linear material. In this respect, two of the most commonly advanced soil models used in practice are:

a) Hyperbolic Soil Model by Duncan & Chang (1970), (called DC model).
b) Hardening Soil Model by Vermeer (1978), Schanz (1998), (called HS model).

Both models use a reference confining pressure to define soil stiffness, however, whereas the DC model uses the atmospheric pressure as a reference (100 kPa, 2.04 ksf), in the HS model the reference confining pressure is a variable. The models are also similar in that they use hyperbolic shear stress-strain formulations (Figure 6). One of the differences between the two models, is that the DC model is defined by the initial stiffness E_i while the HS model uses the secant modulus E_{50} (at 50% of the ultimate stress level). The two stiffness parameters can easily be correlated to each other. A further similarity between the two soil models is they both simulate unloading and reloading with a linearly elastic stiffness response (E_{ur}). In cofferdam excavations, the unloading/reloading modulus is a very important parameter since soils are unloaded laterally and vertically. In this respect, the linear modeling of the unloading/reloading behavior predicts slightly greater unloading and reloading strains.

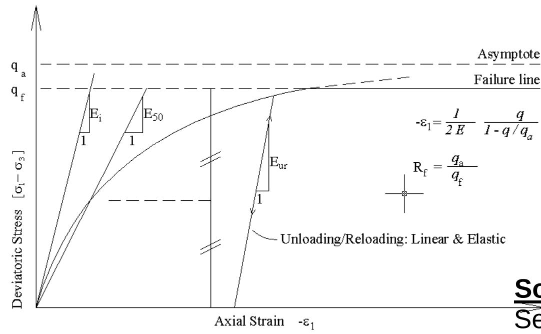


Figure 6: Hyperbolic Stress Strain Modeling

While both models use a hyperbolic loading response, the HS model can capture volume changes associated with dilation during shearing that the DC soil model can not account for. This is achieved by hardening of the yield surface in association with a flow rule that relates volumetric strains to shear strain together with the ultimate dilation angle. The theoretical background is based on the critical state mechanics concept of the constant volume frictional angle that predicts shear resistance at large strains. Strain hardening can also occur during compression loading as shown in Figure 7. The cap surface essentially defines a set of stress path combinations beyond which a soil would experience a loading response (for HS & DC models this would be a hyperbolic behavior). If the stress path retracts within the cap then the behavior will be controlled by the unload/reload response of the soil model (in HS & DC this is a linear elastic response). Beyond the current maximum state of stress experienced by a soil, when compression is increased and the stress path remains within the Mohr-coulomb failure line, the cap expands typically as shown in Figure 7. For both the HS and DC models this cap surface is described by the equation of an ellipse.

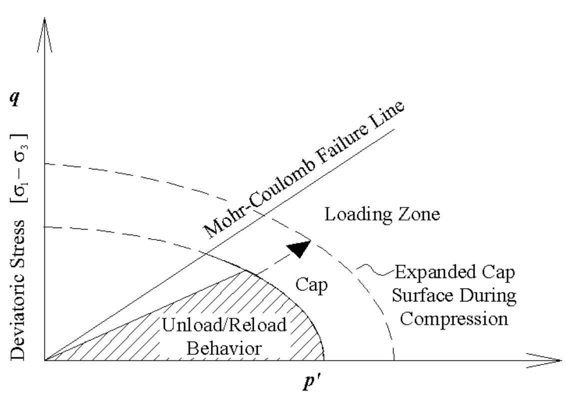


Figure 7: Cap Surface

While the HC and DC models are considered advanced soil models, they both share similar shortcomings. Most significantly, strain softening behavior at very large strains is not captured by any of the two models. Another limitation is that the cyclic loading plastic strains (repeated unloading/reloading between two similar stress conditions) can not be captured.

Soil Tests & Finite Element Parameter Selection

Selecting appropriate engineering properties of soil materials for finite element analyses can be as challenging as performing finite element computations by hand. As designers, we are confronted with a number of parameters that have to be selected. Adding to complexity, soil properties are variable even in laboratory tests and a number of factors can influence test results one way or the other. As a result, considerable judgment is required in selecting appropriate stiffness, strength, and engineering soil parameters.

While there is no single best way to estimate soil parameters, a combination of in-situ and laboratory tests generally provides data to estimate most desired parameters. Table 1 provides a list in-situ and laboratory soil tests that can be used to estimate specific engineering soil parameters. From a quick view of Table 1 one realizes that there is no single test that can cover all soil properties. However, one should use caution with the standard penetration test (SPT). Engineering properties estimated with SPT tests are highly speculative and should not be relied upon.

Table 1: Matrix relating model parameters to soil investigation (CUR 195, 2000)

Soil investigation tests	Cone (piezo) Penetration tests	Dissipation tests during CPT	Pumping test	CPM-tests, Menard test	In-situ Vane test	Dilatometer test	Standard Penetration test	Oedometer test unload reload	Unconfined Undrained	Consolidated Undrained Standard	Consolidated Undrained Extension	Consolidated Undrained Compression	Consolidated Undrained unloading	Consolidated Drained tests	Direct and simple shear tests	Classification tests	Torvane tests	Sieving	Constant and falling head	Unit weight (wet and dry)
Parameters									Triaxial tests											
γ_{dry}	X			X		X	X									√				√
$\gamma_{wet\ (saturated)}$	X			X		X	X									√				√
k_x	X	X	√	X				X										X	√	
k_y	X	X	√	X				X										X	√	
c_k			√					X											X	
e_{init}								X								√		X		
e_{min}																√		X		
e_{max}																√		X		
E_{ref}	X			X		X	X	X		√	√	√	√	√	X	X				
G_{ref}	X			√		X	X	X		X	X	X	X	X	√	X				
E_{oed}	X			X		X	X	√		X		X		X	X	X				
E_{50}^{ref}						X	X	X		√		√		√						
E_{oed}^{ref}						X	X	√												
E_{ur}^{ref}				√							√		√	√						
power (m)										√				√						
R_f										√	√	√	√							
λ^*	X							√								X				
κ^*								√								X				
μ^*								√								X				
K_0^{nc}	X			√		X														
OCR	X			X		X		√								X				
POP								√												
ν				X						√		√		√						
ν_{ur}				X							√		√							
c'	X				√	X			√	√	√	√	√	√	√	X	√			
φ'	X			X		X				√	√	√	√	√	√	X	X	X		X
ψ	X			X						X	X	X	X	√						
max. tensile strength										X			√	X						

√ Means that the model parameter can directly be evaluated from the test.
X Means that the model parameter can be estimated through correlations.

Cone Penetration Tests (CPT) offer a more viable way to obtain engineering parameters, especially when results are calibrated against laboratory tests. CPT can be especially useful in sands that are difficult to sample and test in their natural state without disturbance. For sands, stiffness parameters for sands can be estimated from published charts as shown in Figure 8. It is the author's opinion that a good balance of cone penetration tests and laboratory triaxial shear tests is the best way to estimate soil properties. The author is aware of sites were more than 200 borings with SPT tests have been completed over the last 60 years with no proper soil tests have ever been conducted! When there are no available soil test data then parameters can be estimated from available published information. In this case, performing sensitivity analyses with varying strengths and stiffness scenarios is probably a very good idea.

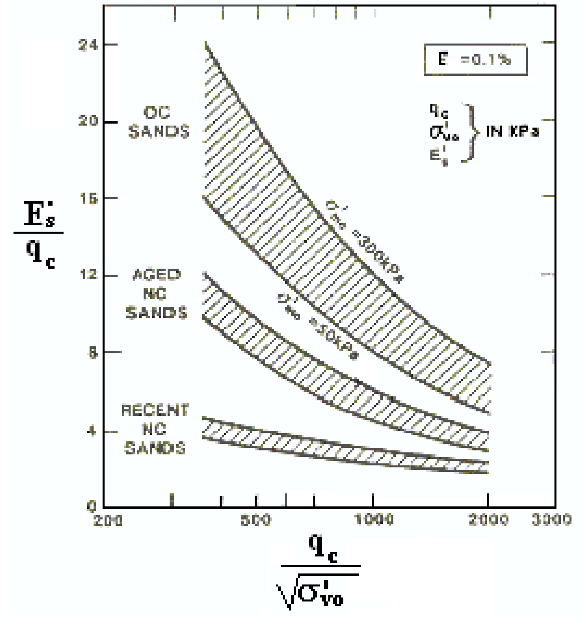


Figure 8: Estimation of secant modulus of elasticity for sands from cone penetration tests (Brinkergreve, 2002)

As discussed before, cohesive soil Figure time-dependent response. For undrained loading, the modulus can be described by either the undrained Young's modulus E_u or the shear modulus G. The shear modulus actually describes the soil "skeleton" response, so it is independent of drainage conditions, all other factors being equal (Kulhawy & Mayne, 1990). Duncan and Buchignani (1976) have compiled a very useful chart for estimating the ratio of undrained secant modulus to undrained shear strength of overconsolidated clays (Figure 9).

In cofferdams soil is primarily unloaded as the excavation progresses. Thus when hyperbolic soil models are used, the predicted behavior is controlled by the linear reload-unload modulus of elasticity (E_{ur}). As a first order estimate E_{ur} can be taken as three to four times the secant modulus of elasticity E_{50} for sands, and as three to five times E_{50} for normally consolidated clays. Over-consolidated clays typically have greater ratios of E_{ur}/E_{50}. E_{ur} should be estimated directly from test data when possible.

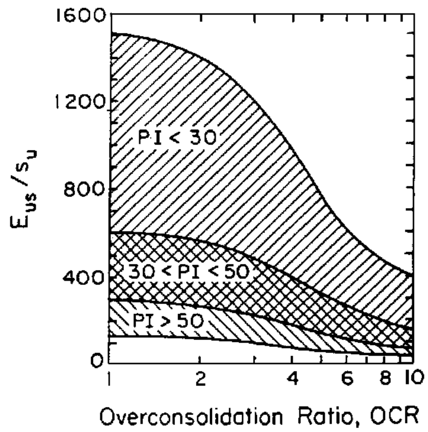


Figure 9: Generalized Undrained Modulus Ratio versus OCR and PI (Duncan & Buchignani, 1976)

Finite element modeling strategy

Formulating an appropriate modeling strategy is essential in generating a finite element model that "reasonably" captures the behavior of a cofferdam excavation. In this respect the GIGO principle applies to all finite element simulations (good input = good output). A good modeling strategy should include some of the following steps:

I. Finite element model layout: Drawing a model simplifies the thinking process and sometimes may reveal mechanisms of failure or other considerations that have to be taken into account. In general, models should extend at least two to three times the excavation depth (or wall depth) beyond the excavation. When basal instability is an issue, the model depth should extend to a stiff boundary (such as bedrock) and include all clay layers that contribute to basal movements.

II. Soil parameters: Appropriate selection of soil strength and stiffness parameters from laboratory or in-situ tests as described in the previous section. When soil tests are absent then parameters can be estimated from published literature as first order estimates.

III. Construction staging: Finite element models of cofferdam excavations should accurately model construction staging by including all stages and construction activities affecting results. Typically, all multi-prop excavations should include a cantilever stage and then subsequent installation of bracing and further excavation until final subgrade is reached. When excavations in clay are modeled, it may be more appropriate to consider construction times between stages in the analysis. In some cases it may be prudent to include ground modifying effects such as jet grout installation and soil loses where appropriate.

IV. 2D vs. 3D: Although the main focus of this paper is on two dimensional analyses, three dimensional effects can be considerable in cofferdam excavations of limited width. While in engineering practice three dimensional effects are typically ignored, one may be able to obtain more economical designs if such effects are included. However, despite the increase in computational power 3D analyses can still be very time consuming.

V. Engineering judgment: Ironically, one of the most important aspects of finite element analyses is the human factor. Once an analysis has been performed, results must be criticized with considerable judgment by a qualified engineer with experience in actual excavation behavior. Performing sensitivity analyses also yields valuable insight. It is always useful to compare finite element results against conventional design methods. In this respect, performing a conventional design bounds the problem and gives indications of expected finite element results.

Case Study analyzed with Finite Elements

A useful case study is presented in this section. The cofferdam studied was constructed within the reservoir pool of the Martin Manatee dam in Manatee, Florida. The dam is 50 ft high earth filled dam with a concrete face. A new intake structure was to be constructed 5 ft from the tail of the dam. This involved the construction of a 58 ft long by 39 ft wide cofferdam with the subgrade nearly 40 ft below maximum water pool elevation (Figure 10). Ground movements caused by excavation and fill placement had to be minimized in order to avoid breaching the dam and causing failure. Support of excavation was provided by three levels of stacked internal bracing that were slid into their design elevations. Excavation was performed in the wet and dewatering took place only after the base slab had been poured and gained sufficient strength.

Soils at the site consist of cemented sands and a softer clay layer at 20 ft below subgrade. Available soil information did not include any laboratory strength tests on the soft clay layer. Hence, soil stiffness parameters had to be estimated using cone penetration data performed on a nearby site using published information (Figure 8).

Initial models indicated lateral soil movements in excess of 1.0 inch towards the excavation. Predicted settlements were even greater due to rip rap being placed on top of the existing dam behind the sheeting. While it was recognized that the problem was inherently three dimensional, the owner was very risk averse. As a result, full three dimensional effects were not incorporated into the finite element analyses. The finite element model showed that replacing the rip rap with a light fill would significantly minimize predicted wall and soil movements (LF in Figure 11).

During construction, the dam was intensively monitored with a number of settlement monitoring points and inclinometers. Practically no soil movements were registered. Predicted lateral sheet pile movements after the base slab had been poured and the cofferdam dewatered were in the order to 0.5 inches. This discrepancy in predictions and measurements can be attributed mainly to three factors: a) three dimensional soil behavior was ignored, b) Sand cementation was ignored, and c) the soft clay layer in all likelihood is not continuous and is interbedded with sand and silt. Figure 12 shows typical soil predicted movement patterns after the excavation has been dewatered. Estimated lateral displacements were in the order of 0.5 inches.

Figure 10: Cofferdam during construction

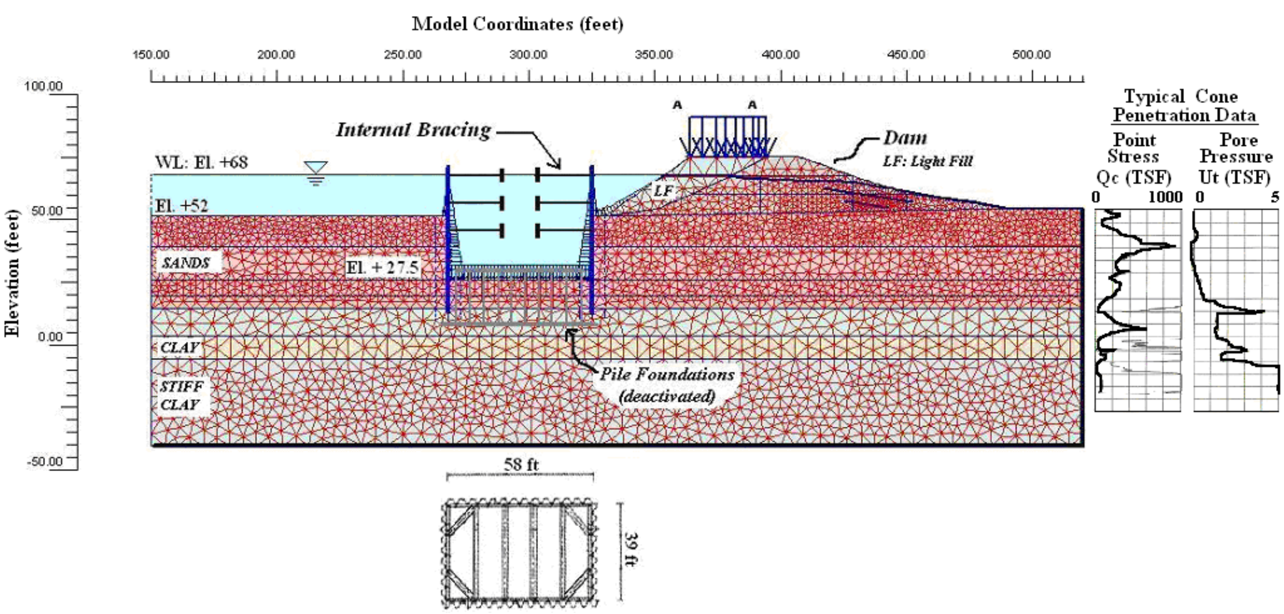


Figure 11: Finite element model at final subgrade

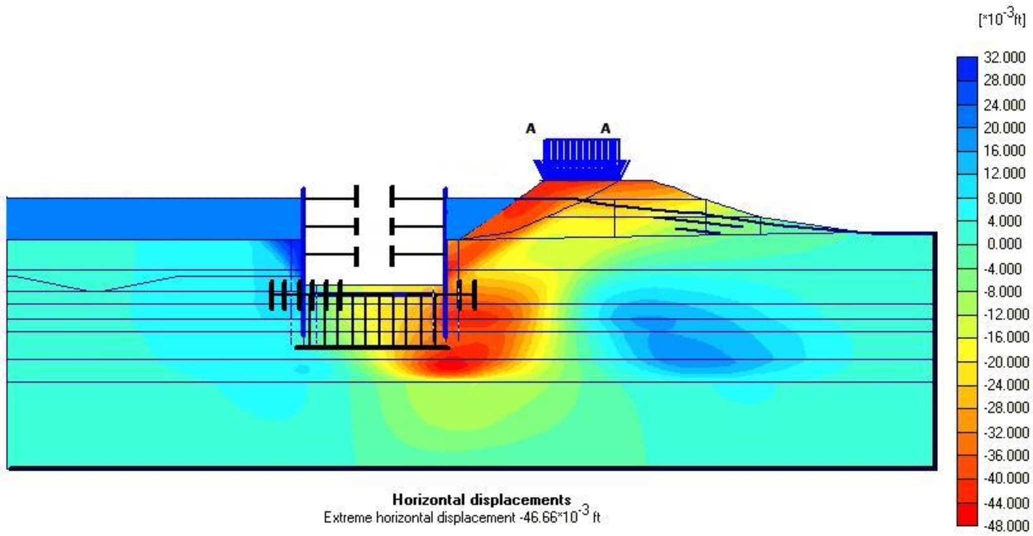


Figure 12: Sample predicted soil displacements with 2D finite element analysis.

Future and Conclusions

While significant progress and achievements have been made in the field of geotechnical finite element mechanics, further advancements are necessary. On the practical side, as computational power increases it is expected that three dimensional software programs will be come increasingly popular and more affordable. On the theory, further research is required to formulate models that capture soil softening, and cycling unload-reload behavior. The continuation of benchmarking efforts is essential to the acceptance and modification of finite element methods in geotechnical engineering.

Not all aspects of finite element modeling have been covered in this article. Numerous publications discuss practical and theoretical aspects in much greater detail. The author encourages the interested reader to further investigate as many relating publications as possible. One should keep in mind that there is no single answer to geotechnical problems and especially finite element analyses. It is almost certain that two different analysts will in all likelihood produce two different results for the same problem. Hence, human judgment is by far the most important aspect of finite element analyses of cofferdam excavations.

References

Brinkergreve, R.B.J., Vermeer. P.A., “PLAXIS – Finite Element Code for Soil and Rock Analyses”, 1998, Users Manual – Version 7 for Windows 95/NT. A.A. Balkema, Rotterdam, The Netherlands

Brinkergreve, R.B.J., “PLAXIS Newsletter No. 12”, 2002, Plaxis B.V., The Netherlands

CUR 195, “Backgrounds on Numerical Modelling of Geotechnical Constructions”, Part 3, Centre for Civil Engineering Research and Codes, April 2000, CUR, Gouda, The Netherlands

Duncan, J.M. and Chang, C. –Y., “Nonlinear Analysis of Stress and Strain in Soils”, Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 96, No. SM5, Sept. 1970, pp. 1629-1653

Duncan, J.M. and Buchignani, A. L., “An Engineering Manual for Settlement Studies”, Department of Civil Engineering, University of California, Berkley, June 1976, 94p.

Kulhawy, F.H. and Mayne, P. W., “Manual on Estimating Soil Properties for Foundation Design”, Cornell University, 1990, Geotechnical Engineering Group, EL-6800, Research Project 1493-6

Skempton, A.W., "The Bearing Capacity of Clays", 1951, Proc. Building Research Congress, vol. 1, pp. 180-189.

Vermeer, P.A., “A Double Hardening Model for Sand”, 1978, Géotechnique 28, No.4, 413 - 433

Whittle, A. J., “Computational Geotechnics; New York ,January 2005”, 2005, Plaxis Course Material, New York, NY.